

A new modified algorithm of NLMS

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Abstract: In this paper, we first study the LMS algorithm, and then study the normalized LMS algorithm. On the basis of the normalized LMS algorithm, a new iterative formula is derived and modified NLMS algorithm, which can overcome the shortcoming of the input signal autocorrelation is too small lead to the step value too large. The simulation results show that the convergence speed of the modified normalized LMS algorithm is faster than that of the LMS algorithm. Moreover, the convergence rate of this method is obviously accelerated with the increase of the step value factor, and the distortion is very small.

Key Words : Adaptive Filter Algorithm; LMS; NLMS;

I. INTRODUCTION

Adaptive filtering algorithm is one of the most active research topics in adaptive signal processing. Adaptive filtering algorithm is widely used in many fields such as system identification, echo cancellation, adaptive spectral line enhancement, adaptive channel equalization, speech linear prediction, adaptive antenna array and so on [1]. In short, the fast convergence rate, low computational complexity and good numerical stability of the adaptive filtering algorithm is the goal of researchers' continuous efforts to pursue. The adaptive filter and the corresponding algorithm have the advantages of simple structure and low computational complexity, which are widely used in practice [2]. The variable step size adaptive filtering algorithm is used to solve the contradiction of the convergence speed and the time-varying tracking speed and the convergence accuracy with μ which is the factor used to adjust the step size. However, the choice of other parameters in the variable step size is also to be determined by experiment, and it is not convenient to use. In this paper, we study the adaptive gradient LMS algorithm and NLMS algorithm. After repeated experiments, the change rule of the gamma parameter is modified. The simulation results show that the modified NLMS algorithm has faster convergence than the LMS algorithm.

II. LMS ALGORITHM

Many scholars have studied the algorithm of reducing the computational complexity and shortening the time of the adaptive convergence [3]. The least mean square algorithm (LMS) is proposed by Windrow et al., Stanford University, which is a method for estimating the gradient vector with instantaneous value. The formula of this method is as follows:

$$\hat{\nabla}(n) = \frac{\partial [e^2(n)]}{\partial w(n)} = -2e(n)x(n) \quad (1)$$

It can be seen that this method is unbiased, because its expected value $E[\hat{\nabla}(n)]$ is equal to the vector $\nabla(n)$. So, according to the relationship between the change of the adaptive filter coefficient vector and the

direction of the gradient vector estimation, the formula of LMS algorithm is as follows:

$$\begin{aligned} \hat{w}(n+1) &= \hat{w}(n) + \frac{1}{2} \mu [-\hat{\nabla}(n)] \\ &= \hat{w}(n) + \mu e(n)x(n) \end{aligned} \tag{2}$$

When the time $n=0$, the filter coefficient vector is used as the arbitrary starting value $w(0)$, and then the calculation steps of the LMS algorithm are as follows:

(1) When the time is n , the filter coefficients vector estimated value is $\hat{w}(n)$, the input signal vector is $x(n)$ and the expected signal is $d(n)$. The error signal calculation formula is as follows:

$$e(n) = d(n) - x^H(n) \hat{w}(n)$$

(2) Using the recursive method to calculate the update estimates of the filter coefficient vector:

$$\hat{w}(n+1) = \hat{w}(n) + \mu e(n)x(n)$$

Increase 1 to the time index n , return to the steps (1), repeat the above steps, until the steady state is reached. Thus, the adaptive LMS algorithm is simple, it not only do not calculate the correlation function of the input signal, but also does not require the inverse of the matrix, so it has been widely used. because the LMS algorithm uses the instantaneous estimation of the gradient vector, it has a large variance, which can't obtain the optimal filtering performance.

III. MODIFIED ALGORITHM OF NLMS

If you do not want to use the correlation matrix of the estimated input signal vector to speed up the convergence rate of LMS algorithm, then the variable step size method can be used to shorten the adaptive convergence process, one of the main method is the NLMS (Normalized LMS, Abbreviated as NLMS) algorithm[4]. The update formula of variable step size $\mu(n)$ is as follows:

$$w(n+1) = w(n) + \mu(n)e(n)x(n) = w(n) + \Delta \bar{w}(n) \tag{3}$$

In the formula, $\Delta \bar{w}(n) = \mu(n)e(n)x(n)$ represents the adjustment of the filter weight coefficient vector [5]. In order to achieve the goal of fast convergence, it is necessary to select the value of variable step size μ , a possible strategy is to reduce the instantaneous square error, that is, using the instantaneous square error as a simple estimate of the mean square error of MSE, which is also the basic idea of LMS algorithm. Instantaneous squared error can be written as follows:

$$\begin{aligned} e^2(n) &= [d(n) - x^T(n)w(n)]^2 \\ &= d^2(n) + w^T(n)x(n)x^T(n)w(n) - 2d(n)w^T(n)x(n) \end{aligned} \tag{4}$$

If the change of the filter weight vector is $\bar{w}(n) = w(n) + \Delta \bar{w}(n)$, then the corresponding square error $\bar{e}^2(n)$ can be obtained by the follow formula:

$$\begin{aligned} \bar{e}^2(n) &= e^2(n) + 2\Delta \bar{w}^T(n)x(n)x^T(n)w(n) \\ &+ \Delta \bar{w}^T(n)x(n)x^T(n)\Delta \bar{w}(n) - 2d(n)\Delta \bar{w}^T(n)x(n) \end{aligned} \tag{5}$$

In this case, the variation of the instantaneous squared error $\bar{e}^2(n)$ can be defined by the following formula:

$$\begin{aligned} \Delta e^2(n) &= \bar{e}^2(n) - e^2(n) \\ &= -2\Delta \bar{w}^T(n)x(n)e(n) + \Delta \bar{w}^T(n)x(n)x^T(n)\Delta \bar{w}(n) \end{aligned} \quad (6)$$

The relationship of $\Delta \bar{w}(n) = \mu(n)e(n)x(n)$ can be substituted into formula (6), we can get the following formula:

$$\Delta e^2(n) = -2\mu(n)e^2(n)x^T(n)x(n) + \mu^2(n)e^2(n)[x^T(n)x(n)]^2 \quad (7)$$

In order to increase the convergence rate, the proper selection of A can minimize the square error, so the formula (7) is used to calculate the partial derivative of the variable coefficient A and make it equal to zero, and the value of A is obtained. this step value $\mu(n)$ leads to a negative value of $\Delta e^2(n)$, which corresponds to the minimum point of $\Delta e^2(n)$, which is equivalent to the Squared error $\bar{e}^2(n)$ equals zero[6]. In order to control the misalignment, taking into account based on the derivative of the instantaneous squared error is not equal to derivative value of mean square error MSE, so update iteration formulas of the LMS algorithm as amended as follows:

$$w(n+1) = w(n) + \frac{\mu}{\gamma + x^T(n)x(n)} e(n)x(n) \quad (8)$$

In the formula, a is fixed convergence factor to control the disorder, B is to avoid the denominator is too small to step value is too large and set the parameters. We say that the formula (8) is the iterative formula of the normalized LMS algorithm. In this paper, the convergence process is regulated by the convergence factor μ , a faster convergence process is obtained by the convergence factor step μ . For the constant value μ , the convergence speed and the offset is a pair of contradictions, to get faster convergence speed can choose large μ , which will lead to a larger offset; To meet the requirements of the offset, the convergence speed is restricted. According to the practice of many experiments, this paper modifies the change rule of parameter γ as follow:

$$\gamma = \mu^{-1} x^T(n)x(n) \quad (9)$$

So as to overcome the shortcomings of $x^T(n)x(n)$ too small to cause the step value is too large.

IV. SIMULATION AND RESULT ANALYSIS

In this paper, MATLAB software is used to simulate the LMS algorithm and the modified NLMS algorithm and its performance. From Figure 1 it can be seen that the error Square mean value of the LMS algorithm decreases with the increase of the sampling points. The convergence of the LMS algorithm is not particularly fast; However, the mean curve of the error square of NLMS algorithm decreases faster with the increase of sampling points, smooth near 100 points, and the convergence of the modified NLMS algorithm is fast, and the mean value of the error square is relatively small.

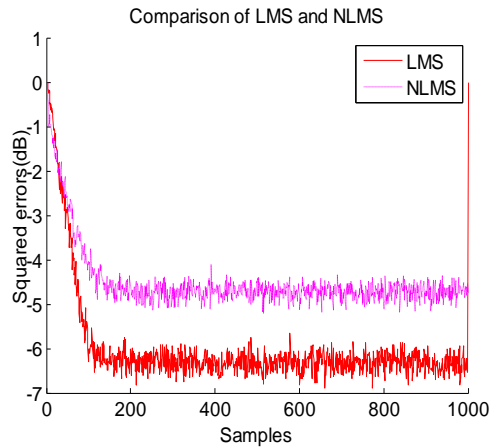


Fig.1 Performance comparison between LMS and the modified NLMS

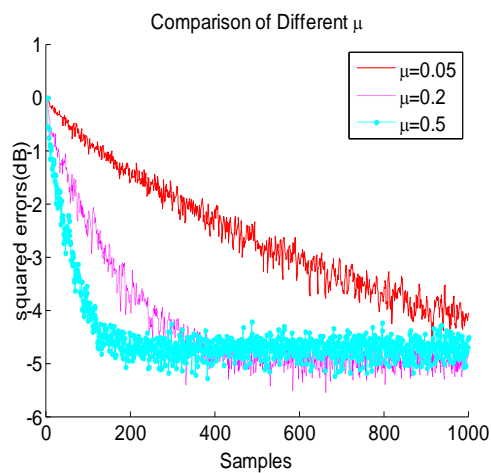


Fig.2 Comparison of the modified NLMS with different μ

Figure 2 is the simulation curve of the modified NLMS algorithm when the step size μ is 0.05, 0.2 and 0.5. From the three curves, we can see that the step size factor μ is smaller, and the convergence speed is slower, and the convergence of the algorithm is accelerated with the increase of the value of μ .

V. CONCLUSION

In this paper, we first study the LMS algorithm, and then study the NLMS algorithm. On the basis of the NLMS algorithm, NLMS is derived and modified by a new iterative formula, which can overcome the shortcoming of $x^T(n)x(n)$ is too small lead to the step value too large. Finally, through MATLAB simulation we can know that the convergence speed of NLMS algorithm is faster than the LMS algorithm, and it also shows that the equivalent step size is the nonlinear variable of the input signal, which makes the variable step size gradually smaller and accelerates the convergence process. When the step size factor μ takes a larger value, the NLMS algorithm not only converges fast but also the distortion of the output waveform after the adaptive filtering is very small. The simulation results show that the modified normalized LMS algorithm converges faster than the LMS algorithm, and with the increase of μ , the convergence rate is accelerated obviously and the distortion is very small; Compared with other adaptive algorithms, the proposed algorithm in this paper has better practicability.

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